Leo-Rey Gordon Publishing leorey1@gmail.com
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The Fiscal Cliff -

Using Simple Models to Demonstrate its Possible Outcomes

Leo-Rey C. Gordon

The United States government's policies of the past two decades have led to an unsustainable path for its federal budget. As of December 2012 total outstanding debt was over $16 trillion dollars, while federal deficits in each of the past four years have exceeded $1 trillion (Budget of the U.S Government, 2011). The federal budget for fiscal year 2013 includes the expiration of various tax and expenditure concessions. This "*fiscal cliff*", a term coined by Ben Bernanke, has created mass uncertainty in markets globally. This commentary uses three economic models to demonstrate one side of the impact of taxes in various markets.[1]

The *fiscal cliff* describes the expiration of the 2001 & 2003 Bush Tax cuts, the two percent pay-roll tax holiday, and exemption packages for miscellaneous tax deductions. Further, additional taxes are set to be levied as a part of the *Patient Protection and Affordable Care Act*. Generally, the expiration of the Bush tax cuts on individual tax payers will result in increases in marginal tax rates, an increase in the capital gains tax, and the lowering of the child care credit. Incentives for capital expenditure deductions will also be removed. While the *Patient Protection and Affordable Care Act* will result in added increases in pay roll taxes, increased medical expense deduction for certain limits, plus government fess on medical material and device manufacturers.[2] The Congressional Budget Office has estimated that the *Fiscal Cliff* will reduce the Federal deficit by 5.1 percent, leading to an economic contraction of 1.3% in the short-run (Congressional Budget Office, 2012). The following sections will use economic models as a framework to demonstrate outcome possibilities in various markets.

A Partial Equilibrium Model of Demand and Supply

The quantity of goods demanded in the market, (Q_d), at a given level of income, can be represented as a function of the price of goods (P) and the tax rate on income (t).

[1] As with all economic phenomena there are contradicting outcomes that can also be described.
[2] See The Fiscal Cliff: A Primer (2012) for a detail review of all amendments.

$$Q_d = D(P, t) \qquad (1)$$

By the economic law of demand, the quantity demanded for normal goods by the rational consumer, will fall as the price level and rate of tax increases. This law is reflected such that the derivative of the quantity demanded with respect to price, and the derivative of the quantity demanded with respect to a change in the tax rate, are less than zero.

$$dQ_d/_{dp} < 0 \quad \& \quad dQ_d/_{dt} < 0 \qquad (2)$$

Likewise, the quantity of goods supplied by firms, at a given level of costs, can be represented as a function of the selling price and the corporate tax rate (t_c).

$$Q_s = S(P, t_c) \qquad (3)$$

Similarly, by the economic law of supply, firms will increase the supply of goods and services at higher prices, ceteris paribus. But will lower the quantity supplied at higher tax rates.[3] These laws are reflected mathematically as:

$$dQ_s/_{dp} > 0 \quad \& \quad dQ_s/_{dt_c} < 0 \qquad (4)$$

Based on these laws and under the requirement of a market clearing equilibrium in the goods market, how will small changes in tax rates impact the equilibrium levels of prices and output in the market? Now recall that the total differential of a mathematical function provides the total effect on the left hand side variable if all right hand side variables change by some small amount. The total differential can then be used to represent the change in demand expected if prices and taxes change by a small amount. Likewise, the total differential of the quantity supplied will represent the change in the quantity supplied given small changes in the price and corporate tax rate.

The total change in the quantity demanded (total differential) is the change in the quantity demanded given a change in the price level multiplied by the change in price, plus the change in the quantity demanded given a change in the tax rate multiplied by the change in the tax rate.

[3] For both demand and supply, the relationship with the tax rate can be more accurately presented as the relationship with disposable income in the case of demand, and net profits in the case of supply. Higher tax rates, reduces both incomes, and therefore is inversely related with the quantity supplied and demanded.

Similarly, the total change in the quantity supplied is the change in the quantity supplied given a change in the price level multiplied by the change in price, plus the change in the quantity supplied given a change in the tax rate multiplied by the change in the tax rate. Both total changes are represented in the equations below.

$$dQ_d = {}^{dQ_d}/_{dp} * dp + {}^{dQ_d}/_{dt} * dt \qquad (5)$$

$$dQ_s = {}^{dQ_s}/_{dp} * dp + {}^{dQ_s}/_{dt_c} * dt_c \qquad (6)$$

Now market equilibrium is defined as the condition in which the quantity of goods demanded in the market is just equal to the quantity supplied. When this condition holds, there are no shortages or surpluses in the market, no readjustments are made, and therefore prices and output will remain stable. For any change in the quantity demanded or supplied, markets will clear, and economic equilibrium maintained once the total change in the quantity demanded is matched by an equivalent change in that supplied. That is:

$$dQ_d = dQ_s \qquad (7)$$

This market clearing condition can be used to demonstrate the market effect of a change in consumer taxes. Assume that the change in corporate tax is zero, the market clearing requirement (7) becomes:

$${}^{dQ_d}/_{dp} * dp + {}^{dQ_d}/_{dt} * dt = {}^{dQ_s}/_{dp} * dp \qquad (8)$$

Re-arranging the subject of the formula and then simplifying gives:

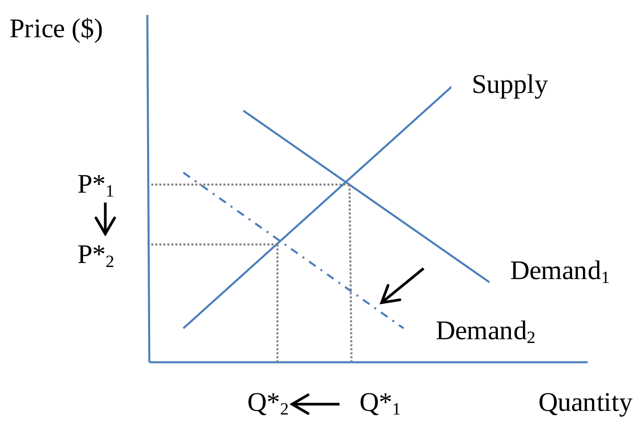
$$\left({}^{dQ_d}/_{dp} - {}^{dQ_s}/_{dp}\right) * dp + {}^{dQ_d}/_{dt} * dt = 0 \qquad (9)$$

The subject of the formula can further be changed to obtain the market clearing price given a change in the tax.

$${}^{dp}/_{dt} = \frac{{}^{dQ_d}/_{dt}}{{}^{dQ_s}/_{dp} - {}^{dQ_d}/_{dp}} \qquad (10)$$

Based on the law of supply and demand represented by conditions 2 & 4, the denominator of (10) is positive, while its numerator is negative. This implies that market clearing prices will decrease given some increase in the individual tax rate. This result is represented graphically below. Here an increase in the individual payroll tax reduces disposable income, resulting in a downward shift of the consumer's demand curve[4]. At the original price P^*_1 there are surpluses in the market. Producers are forced to reduce prices while consumers lower their willingness to pay.

The Goods Market



These market forces result in a new lower market clearing price of P^*_2, such that less output is being sold at lower prices.

A Labor-Leisure Model of Labor Supply

A common debate associated with the Fiscal Cliff and increases in labor taxes in general, is regarding the effect of taxes on labor hours and productivity. The issue often discussed is to what extent a lower after tax wage creates a disincentive to work. This issue will be explored using a labor leisure model of labor supply.

Individuals aim to maximize their personal satisfaction, or utility (U), which can be represented as a function of the amount of goods and services consumed (c), as well as the amount of time used for leisure (l). The other use of time is for market work (m) which provides wages (w) to be

[4] A partial equilibrium model of the labor market could also be used to demonstrate the effect in the labor market of increased taxes. It is the reduction in after tax wages, which reduces the consumer demand.

used for the purchasing of consumption goods. An individual's decision can then be modeled as a constrained optimization problem of maximizing their personal satisfaction subject to the requirement that the value of consumption must equal all labor and non-labor income[5].

$$Max: U(c, l) \qquad (11)$$

$$\text{subject to: } p * c = w * m + v \qquad (12)$$

$$l + m = T \qquad (13)$$

Here, p represents the price of consumption goods, w represents the wage for market work, and v represents exogenous income. This exogenous income could be considered unemployment benefits, or some amount of welfare support given some threshold level of w*m. The second constraint equation (13) shows that leisure time and market work must sum to equal total time in a day (T).

The constrained optimization problem can be represented and solved using the Lagrangian multiplier method[6]. Using this method, the individual's decision problem is then presented as:

$$L = U(c, l) + \lambda(p * c - (w * m + v)) \qquad (14)$$

Each individual optimizes their decision function with respect to consumption and leisure. We therefore re-state the Lagrangian function with respect to these two decisions by substituting market work, m, with equation (13). The individual's problem is now purely reflected by their decision variables, consumption and leisure, as equation (14) now becomes[7]:

$$L = U(c, l) + \lambda(p * c - (w * (T - l) + v)) \qquad (15)$$

The maximum of any continuous function can be found at the point at which its derivative is zero. The individual optimization with respect to consumption is maximized when the derivative of the Lagragian function with respect to consumption is equal to zero

$$dL/_{dc} = dU(c^*, l^*)/_{dc} + \lambda p = 0 \qquad (16)$$

[5] We will ignore the possibility of savings in the individual's decision.
[6] For a review of the use of the Lagrange Multiplier in solving optimization problems see Nicholson (2002).
[7] Recall that the variable T is a constant equal to 24 hours.

Similarly, the level of leisure that maximizes satisfaction is maximized when the derivative of the Lagrangian function with respect to leisure is equal to zero.

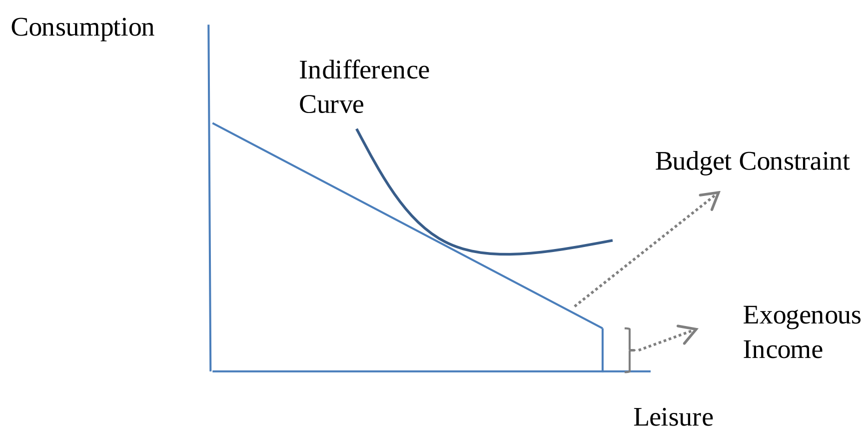
$$dL/_{dl} = {}^{dU(c^*,l^*)}/_{dl} + \lambda w = 0 \qquad (17)$$

Combining these two requirements provides the individual's utility maximizing criteria with respect to consumption and leisure, and is presented in equation (18). The results of the model show that personal satisfaction is maximized only when the marginal rate of substitution between consumption and leisure is just equal to the ratio of work wage to prices. That is:

$$\frac{dU(c^*,l^*)/_{dc}}{dU(c^*,l^*)/_{dl}} = \frac{p}{w} \qquad (18)$$

Intuitively this condition implies that utility is maximized only at the point where the marginal satisfaction obtained from additional leisure, per wage dollar, is just equal to the marginal satisfaction obtained from additional consumption, per dollar of consumption. In other words the rational individual should increase their leisure time as long as the enjoyment obtained outweighs that lost from consumption. This result is also represented graphically below.

Labor-Leisure Model

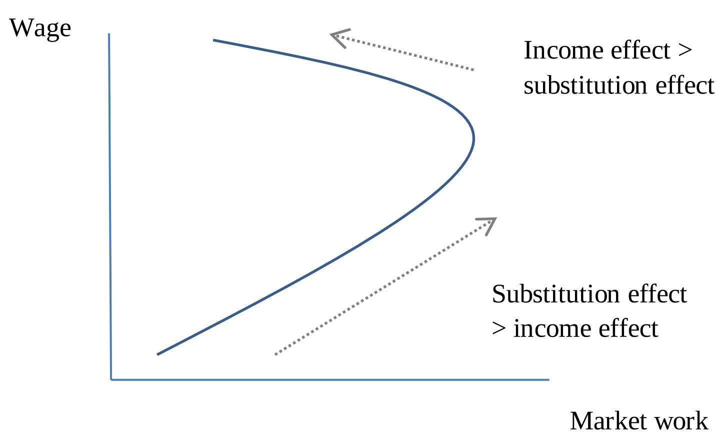


From the budget constraint presented in equation (12) we can derive the slope of the line as w/p. The indifference curve shows the combinations of leisure and consumption that provide the same level of satisfaction. The individual's objective is to attain the highest possible level of personal satisfaction, given their budget constraint. This maximum is attained when the budget is exhausted, also represented at the point at which the slope of both functions are equal. As a result the optimal amount of leisure can be expressed as a function of the slope of the budget constraint, wages and prices, and the level of exogenous income.

The aim of this section of the commentary is to demonstrate the effect that individual taxes has on the supply of labor hours and market work. The overall impact of which depends on the relative magnitude of the *"income effect"* and *"substitution effect"*. Both of these effects create opposing outcomes on labor supply.

The substitution effect is defined as the outcome in which a fall in wages implies that the opportunity cost of leisure falls, this subsequently results in individuals working less, substituting away from consumption, and increasing leisure time. The income effect describes the scenario in which a fall in the take-home wage equates to a lowering of income, which implies that more market work is required to enjoy the same level of consumption. Both the substitution and income effect occur simultaneously but in opposing directions, therefore the effect on labor supply as a result of increases in taxes is uncertain.

A Backward Bending Labor Supply Curve



The figure above demonstrates a backward bending labor supply curve. The idea of a backward bending supply curve will be used to help create expectations on the likely labor supply outcome from the *fiscal cliff*. A backward bending labor supply curve is created due to changes in the relative magnitude of the substitution and income effects based on an individual's wage bracket[8]. The derivation of the backward bending labor supply curve is based on the law of diminishing marginal benefit. That is, at low levels of consumption, additional consumption will provide large increases in personal satisfaction. This effect diminishes as the level of consumption increases. Such that at higher levels the additional satisfaction gained from further consumption increases becomes small.

This law of diminishing benefit thus implies that at low levels of wages, the marginal satisfaction from consumption is high, such that increases in the wage rate provides the incentive to provide more labor hours and reduce leisure. Here the substitution effect is larger than the income effect of wages. At high wage levels, the income effect from increased wages is greater than the substitution effect. Such that for high wage earners the marginal personal satisfaction improvement from consumption is low relative to the marginal benefit from leisure. Thus further increases in wages create an incentive to reduce market work. Recall, that the income effect from increased wages is that individuals now have to work less to maintain the original level of consumption.

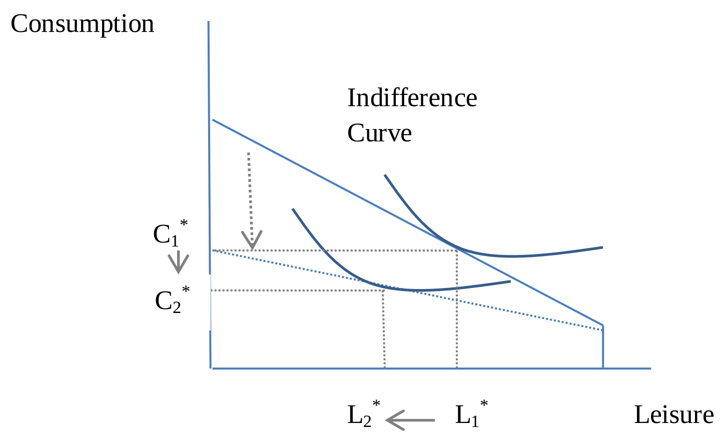
The backward bending labor supply diagram shows that an increase in the tax rate that reduces take home wages will result in an increase market work for high wage earners and a reduction in market work for low wage earners. Given the current direction of fiscal cliff debates, and the general composition of the U.S labor market, I expect than an increase in the tax rate will bring about an overall increase in the supply of market work, that is to say that the aggregate income effect is larger than the substitution effect.

Now based on the previously described labor leisure model of market work, an increase in tax rates will lower take home wages, thus reducing the slope of an individual's budget curve. The level of personal satisfaction attainable will therefore fall. We also see that the increase in the individual tax rate will reduce the personal satisfaction maximizing level of consumption and

[8] I will postpone the mathematical derivation of the backward bending supply curve, but provide a brief description and graphical representation of the concept.

leisure. The diagram below presents this described market outcome, after further assuming that the income effect will dominate the substitution effect on aggregate.

Labor-Leisure Model after taxes



Model of Investment (Tobin's Q)

A significant portion of the fiscal sustainability plan is the removal of tax incentives for capital investment. Currently, there are provisions that allow firms to almost completely immediately deduct certain capital depreciation expenses. Further, there are proposals of increasing the tax rate on capital gains. At the macro level, firm investment demand coupled with household savings determines the level of long term economic productivity. Therefore these adjustments to the timing of costs to capital investment create changes to the value of capital investment, market clearing interest rates and asset prices, and subsequently aggregate business investment. A model of firm investment based on Tobin's Q theory will be used as an outline to discuss one possible effect of the removal of these tax incentives. More detailed description of these models can be found in Tobin (1969), Abel (1982), Hayashi (1982) and Romer (2001).

Firm profit is determined by its capital expenditure decisions and the average level of capital in the industry. Assuming constant returns to scale in firm expansion, within a competitive market holding the proportions of all inputs constant, profits will increase with capital investment. In other words, profits will be generated once capital expenditures increases output by some factor greater than the increase in cost. Further there are additional costs associated with capital expenditures which we will term adjustment costs. These adjustments costs could include among others lost production during periods of transition, or employee search and training costs. The firms profit maximizing decision can then be represented as:

$$\pi = fn(\, k_t) \qquad (19)$$

The model defines q as the present value of cash flows generated from additional capital stock. Such that if q is large, the the firm will undertake further capital investment. The firms' profit maximizing solution occurs when the cost of acquiring capital is just to the marginal value added. Further, since q represents the marginal profits gained from additional investment, in a competitive market with full information, q will also represent the market value of a unit of capital. Through profit optimization with respect to capital investment, the profit maximizing level of capital occurs once condition (20) holds.

$$\pi'(\, k_t) = rq_t - \Delta q_t = 0 \qquad (20)$$

Or

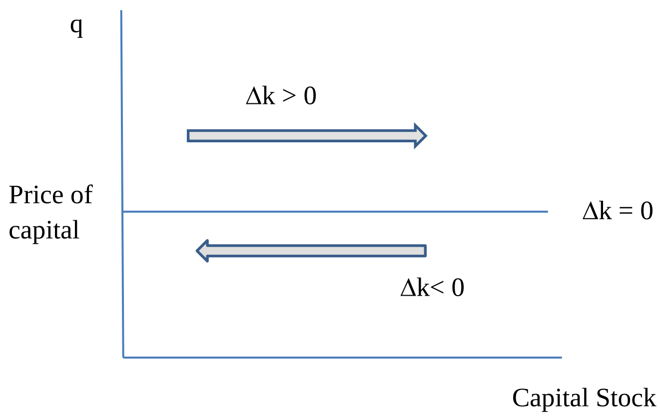
$$rq_t = \Delta q_t \qquad (21)$$

That is, profit maximization occurs when the opportunity cost of capital investment, rq_t, is just equal to return from additional capital investment[9]. A complete aggregate investment decision can be developed by obtaining points at which the profit optimization decision gives no change in capital stock, as well as no change in the value of capital investment. Without providing a mathematical exposition Tobin's Q theory of capital investment is demonstrated graphically in the figures below.

The line $\Delta k=0$ shows the relationships between the value of capital and the stock of capital, at which the change in capital, that is investment, is zero. The change in capital stock is zero once the price of capital is just equal to the present value of the value added, Tobin's q. If q is greater than the price of capital, then there is an incentive for investment and $\Delta k>0$. Likewise if q is less than the price of capital, then there should be liquidation and $\Delta k<0$.

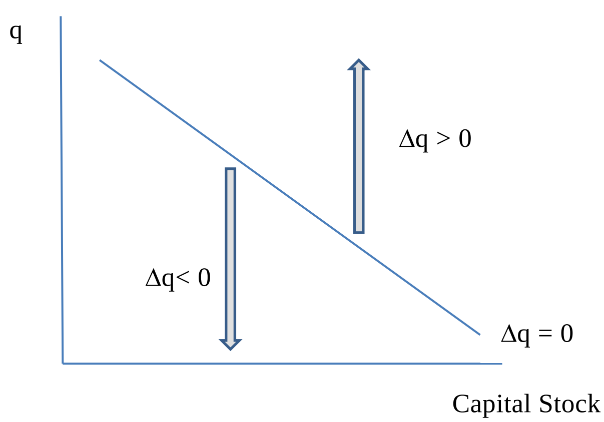
[9] The interest rate of savings is r. Therefore since q also represents the value of capital, the r*q represents forgone interest earnings.

Zero Change in Capital Stock



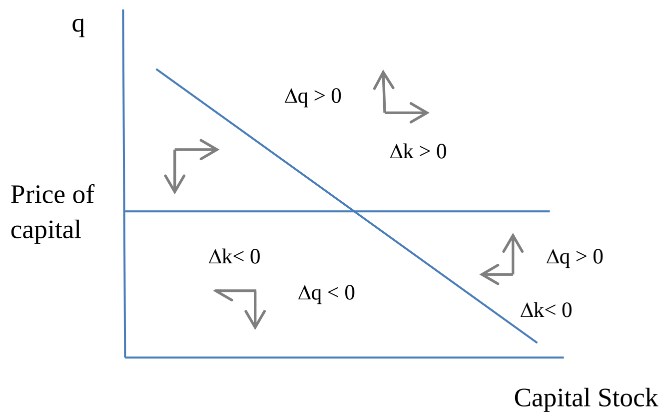
Similarly the model identifies a function between capital and q that results in no change to the value added of additional investment, Δq=0. The figure below shows that if capital stock is to the right of the function then this will result in an increase in the value of additional capital investment. Similarly levels of capital stock to the left of the function will result in the lowering of its market value.

Zero Change in Tobin's q



We can combine both functions to display the phase diagram of Tobin's q theory of investment. Using the above described simplified model, the q value of investment adjusts to direct aggregate capital investment to the stable saddle path. Along this path of capital investment long term equilibrium is achieved.

Phase Diagram of Tobin's Q Theory of Investment



Now the effect of the expirations of capital investment tax incentives corresponds to an increase in the price of capital. This results in a shift upward of the Δk=0 line. With this shift, the original level q will become less than the price of investment capital. This will result in reductions in capital stock. Further, as aggregate levels of capital fall, due to diminishing marginal returns, there will be increases in the value of additional capital, q. A new long run equilibrium in the capital investment market is then obtained at lower levels of aggregate capital, and higher values of q.

This commentary used three economic models to demonstrate market possibilities formed from increases in upcoming tax hikes. The goods market will see a lowering of market prices and less output. The labor market will demonstrate more market work, less leisure and consumption. The capital markets will see less new capital investment and increases in the value of capital investment. A separate commentary will be used to outline the antithesis of these possibilities.

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